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<u>Total Charge</u>

Q: If we know charge density $\rho_v(\bar{r})$, describing the charge distribution throughout a **volume** V, can we determine the **total charge** Q contained within this volume?

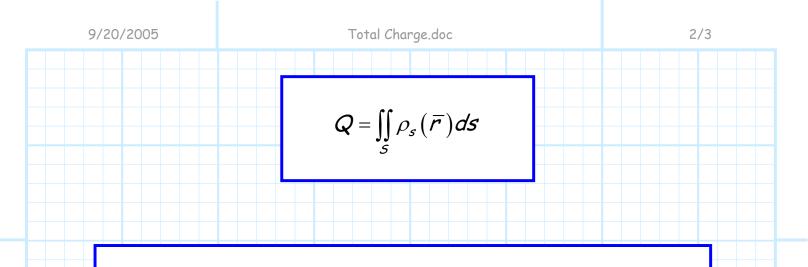
A: You betcha! Simply integrate the charge density over the entire volume, and you get the total charge Q contained within the volume.

In other words:

$$Q = \iiint_{v} \rho_{v}(\bar{r}) dv$$

Note this is a **volume integral** of the type we studied in Section 2-5. Therefore select the differential volume dv that is appropriate for the volume V.

Likewise, we can determine the total charge distributed across a **surface** S by integrating the surface charge density:



Q: Hey! This is **NOT** the surface integral we studied in Section 2-5.

A: True! This is a scalar integral; sort of a twodimensional version of the volume integral.

The differential surface element *ds* in this integral is simply the **magnitude** of the differential surface vectors we studied earlier:

For example, if we integrate over the surface of a sphere, we would use the differential surface element:

$$ds = \left| \overline{ds_r} \right| = r^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

Finally, we can determine the total charge on **contour** C by integrating the **line charge density** $\rho_{\ell}(\overline{r})$ across the entire contour:

$$Q = \int_{C} \rho_{\ell}(\bar{r}) d\ell$$

The differential element $d\ell$ is likewise related to the differential displacement vector we studied earlier:

$$d\ell = \left| \overline{d\ell} \right|$$

For example, if the contour is a circle around the z-axis, then $d\ell$ is:

$$d\ell = \left| \overline{d\phi} \right| = \rho \, d\phi$$